Viscoplastic Bifurcation Buckling of Plates

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A method is proposed according to which the bifurcation buckling load of inelastic rate-sensitive plates subjected to specified boundary conditions can be deduced from the solution of the corresponding perfectly elastic problem under the same boundary conditions. Applications of the method are given for the determination of the buckling stresses of simply supported viscoplastic plates for various values of loading rates and thermal conditions. Both classical and higher-order theories of plates are used in this investigation, and the effect of shear deformation is studied. The numerical results are given for a viscoplastic material that is modeled by the Bodner-Partom unified theory.

Introduction

THE determination of the bucking load of perfectly (i.e., without inelastic effects) elastic structures has received considerable attention (see, for example, Ref. 1 for a recent review). The stability analysis of inelastic plates is much more complicated due to the inherent nonlinearity of the material constitutive law. For total deformation plasticity theory, numerous investigations on buckling analysis can be found in the literature (see, for example, Ref. 2).

It is well known, however, that incremental plasticity theories are more realistic in the modeling of the behavior of inelastic materials. In the framework of the classical theory of incremental plasticity, Hendelman and Prager[3] determined the critical stress of buckling of plates. This work was discussed by Pearson[4] who made some comparisons with experimental results. Hutchinson[5] studied the effect of initial small imperfections on the inelastic buckling of plates and shells, while shear deformation effects were investigated by Shrivastava.[6] An extensive survey and bibliography on time-independent plastic buckling was given by Bushnell.[7]

Recently, the influence of material rate sensitivity on the elastic buckling of an eccentrically stiffened panel was investigated by perturbation and numerical analysis by Tvergaard.[8] For shells of revolution, this effect was investigated by Bodner and Naveh[9] using the finite element method, and they concluded that viscoplastic material behavior could have an important influence on the buckling of structures in the inelastic range.

In the present paper, a method is proposed for the stability analysis of viscoplastic plates. The method is based on the solution of the corresponding perfectly elastic problem that is used in deducing the buckling state. The inelastic plate is initially isotropic, but at later stages of loading plastic deformation develops that leads to the appearance of instantaneous anisotropic properties. It is shown that if the buckling solution of the corresponding perfectly elastic anisotropic plate is known, one can readily determine whether or not inelastic buckling occurs at the current instant.

The present investigation is concerned with the determination of the critical state that leads to bifurcation of the equilibrium path of a given inelastic plate. An instability criterion is established for the prediction of this bifurcation point. The condition for the occurrence of the bifurcation is the loss of out-of-plane stiffness caused by in-plane loading. This loss of stiffness can be expressed by the fact that the stiffness matrix of structure becomes singluar.

In the present paper, the rate-sensitive inelastic behavior of the material is modeled by the unified elastic-viscoplastic theory of Bodner and Partom.[10] This theory does not require a yield criterion or loading or unloading conditions. In all stages of loading history, the material constitutive law involves both elastic and plastic components, but the latter is negligibly small when the material behavior should be essentially elastic. The proposed method of prediction of the bifurcation buckling of viscoplastic structures is illustrated in the cases of plates made of rate-sensitive as well as rate-independent materials. To this end, two types of materials were chosen: commercially pure titanium and aluminum alloy. The first one is highly rate sensitive whereas the latter is weakly rate sensitive (at room temperature). Both classical plate theory (CPT) and higher-order shear deformation theory (HSDT) are used in the buckling analysis of the inelastic plates. In the latter type of plate theory, the shear deformation effects are parabolically distributed across the thickness of the plate.

Illustrations are given that exhibit the effect of applied loading rate, material rate sensitivity, elevated temperature, and relative thickness on the bifurcation buckling of the inelastic plates.

Constitutive Relations

Let us consider an elastic-viscoplastic material. As usual, the total strain rate of the material is taken to be the sum of the reversible (elastic) and irreversible (plastic) strain rate components:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{(E)} + \dot{\epsilon}_{ij}^{(P)} \tag{1}$$

The plastic component of the strain rate is assumed to be given by Prandtl-Reuss flow law as follows:

$$\dot{\epsilon}_{ij}^{(P)} = \Lambda s_{ij} \tag{2}$$

where s_{ij} is the deviatoric part of stress σ_{ij}

$$s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3 \tag{3}$$

where Λ is the flow rule function of the adopted viscoplastic constitutive model and δ_{ij} is the Kronecker delta. Plastic incompressibility is assumed, i.e., $\dot{\epsilon}_{kk}^{(P)} = 0$.

The stress rate $\dot{\sigma}_{ij}$ is related to the elastic components of the total strain rate by the time derivative of Hooke's law

$$\dot{\sigma}_{ij} = [\mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda\delta_{ij}\delta_{kl}]\dot{\epsilon}_{kl}^{(E)} \tag{4}$$

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where μ and λ are Lamé constants of the material.

From the flow law, Eq. (2), we have

$$\dot{\epsilon}_{ij}^{(P)} = \frac{3}{2}\frac{s_{ij}}{\sigma_{EQ}}\dot{\epsilon}_{EQ}^{(P)} \tag{5}$$

where the equivalent stress σ_{EQ} and the equivalent plastic strain rate $\dot{\epsilon}_{EQ}^{(P)}$ are defined by

$$\sigma_{EQ} = (^3/_2\, s_{mn}s_{mn})^{1/2} \tag{6}$$

and

$$\dot{\epsilon}_{EQ}^{(P)} = (^2/_3\,\dot{\epsilon}_{ij}^{(P)}\dot{\epsilon}_{ij}^{(P)})^{1/2} \tag{7}$$

Substituting Eq. (1) into (4), multiplying the result by $\partial\sigma_{EQ}/\partial\sigma_{ij}$, and using the definition, Eq. (6), of the equivalent stress, the following equation is obtained

$$\dot{\epsilon}_{EQ}^{(P)} = \frac{3\mu}{E_P + 3\mu}\frac{s_{kl}}{\sigma_{EQ}}\dot{\epsilon}_{kl} \tag{8}$$

where the "plastic tangent modulus" is defined by

$$E_P = \dot{\sigma}_{EQ}/\dot{\epsilon}_{EQ}^{(P)} \tag{9}$$

Equations (5) and (8) provide

$$\dot{\epsilon}_{ij}^{(P)} = \frac{\mu}{E_P/3 + \mu}\frac{s_{ij}s_{kl}}{s_{mn}s_{mn}}\dot{\epsilon}_{kl} \tag{10}$$

Equations (1), (4), and (10) yield the following constitutive relation for the material:

$$\dot{\sigma}_{ij} = \left[\mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda\delta_{ij}\delta_{kl} - \frac{2\mu^2}{E_P/3 + \mu}\cdot\frac{s_{ij}s_{kl}}{s_{mn}s_{mn}}\right]\dot{\epsilon}_{kl} \tag{11}$$

Or, in a compact form,

$$\dot{\sigma}_{ij} = C_{ijkl}^{VP}\dot{\epsilon}_{kl} \tag{12}$$

where the instantaneous viscoplastic moduli C_{ijkl}^{VP} are defined in Eq. (11).

The plastic tangent modulus E_P in Eq. (11) depends on the loading history and the current state of stress at each loading stage. This function is determined from the adopted viscoplastic model that specifies the explicit form of the flow rule function Λ in Eq. (2). From Eqs. (6), (7), and (9) we obtain

$$E_P = 3\mu\frac{s_{mn}\dot{\epsilon}_{mn}^{(E)}}{s_{pq}\dot{\epsilon}_{pq}^{(P)}} \tag{13}$$

From Eq. (13), the instantaneous moduli C_{ijkl}^{VP} take the form

$$C_{ijkl}^{VP} = \mu(\delta_{ik}\sigma_{jl} + \delta_{il}\delta_{jk}) + \lambda\delta_{ij}\delta_{kl} - 2\mu\frac{s_{rt}\dot{\epsilon}_{rt}^{(P)}}{s_{pq}\dot{\epsilon}_{pq}}\cdot\frac{s_{ij}s_{kl}}{s_{mn}s_{mn}} \tag{14}$$

It can be readily seen that the reduction in material stiffnesses, caused by plastic flow, is a function of the current state of loading and the ratio of the plastic work rate $s_{rt}\dot{\epsilon}_{rt}^{(P)}$ to the rate of the total work of deformation $s_{pq}\dot{\epsilon}_{pq}$, which does not include volume changes. For time-independent plasticity, similar reduction in the material stiffness has been discussed by Hill[11] and Hutchinson[5] using the yield function concept. In the present work, on the other hand, the instantaneous constitutive relation, Eq. (12), in conjunction with Eq. (14) for viscoplastic materials has been obtained without the use of a yield function. This enables the utilization of both the conventional plasticity theories with the yield surface concept as well as unified plasticity theories in which no yield criterion is employed.

It should be mentioned that Eq. (12) resembles the generalized Hooke's law of perfectly elastic anisotropic materials. The present derivation differs, however, in that the instantaneous moduli of "elasticity" [given by Eq. (14)] are functions of stress state and loading history, i.e., they are time dependent. Nevertheless, due to the incremental nature of the viscoplastic process, one can take the instantaneous moduli to be constants during each time increment. This permits the application of an analysis designated to an appropriate elastic problem to the corresponding initially isotropic viscoplastic one.

This methodology can be adopted for the treatment of the buckling of viscoplastic structures. It should be noted, however, that, due to this stepwise linearization, it will be generally impossible to obtain a close-form expression for the buckling load. To this end, the following procedure can be applied for the determination of the buckling stress of a viscoplastic structure. 1) Apply an increment of loading and compute the reduction in material stiffness. 2) By using the instantaneous moduli C_{ijkl}^{VP} (which depend on the present stress state and history), check that at the present stage (increment) of loading the governing equation for the elastic buckling problem is satisfied. This is performed by replacing the elastic stiffness moduli C_{ijkl} [given by Eq. (4)] by the corresponding instantaneous viscoplastic (reduced) moduli C_{ijkl}^{VP} [given by Eq. (14)]. 3) If the answer is positive, then the critical load is achieved. Otherwise, the applied load must be increased, and the above procedure is repeated.

It should be noted that Hendelman and Prager[3] derived the buckling stress of rate insensitive elastic-plastic simply supported compressed plate directly, without using the critical stress of the corresponding anisotropic perfectly elastic plate. The present approach coincides in this special case of rate independent material with the derivation of Hendelman and Prager. However, by using the present methodology of extracting the buckling load of a viscoplastic plate from the corresponding elastic solution, it is possible to solve various problems of viscoplastic plates under different boundary conditions.

In the present paper, the inelastic behavior of materials is represented by the unified viscoplastic theory of Bodner and Partom.[10] This theory does not assume the existence of a yield condition that eliminates the need to specify loading or unloading criterion, and the same equations can be directly used in all stages of loading and unloading. According to these equations, plastic deformation always exists, but it is negligibly small when the material behavior should be essentially elastic.

The flow rule function Λ in Eq. (2) is given according to this theory as follows:

$$\Lambda = D_0 \exp\{-\tilde{n}[Z^2/(3J_2)]^n\}/J_2^{1/2} \tag{15}$$

where $\tilde{n} = 0.5(n+1)/n$ and $J_2 = s_{ij}s_{ij}/2$ is the second invariant of the stress deviator s_{ij}; D_0 and n are inelastic material parameters; and Z is a state variable that represents the hardened state of the material with respect to resistance to plastic flow. In the case of isotropic hardening, the evolution law of this variable is given by

$$\dot{Z} = m(Z_1 - Z)\dot{W}_P/Z_0 \tag{16}$$

where Z_0, Z_1, and m are additional inelastic parameters of material. Note that the plastic work rate $\dot{W}_P = \sigma_{ij}\dot{\epsilon}_{ij}^{(P)}$ is taken as the measure of hardening. The physical significance of the above inelastic constants is as follows. The parameter D_0 is the limiting strain rate, Z_0 is related to the "yield stress" of a uniaxial stress-strain curve of the material, and Z_1 is proportional to the ultimate stress. The material constant m determines the rate of work hardening, and the rate sensitivity is controlled by the temperature-dependent parameter n.

It should be mentioned that in the framework of his theory, an explicit expression for the plastic tangent modulus E_P [Eq. (9)] has been derived by Bodner,[12] in terms of the time-dependent state variable $Z(t)$.

Buckling of Viscoplastic Plates

The bifurcation analysis of an initially isotropic viscoplastic plate is implemented by adopting the incremental procedure for instantaneous anisotropic behavior as described above. Therefore, let us briefly consider in this section some existing buckling solutions for linearly elastic anisotropic plates.

Consider a rectangular elastic anisotropic plate with planform dimensions a and b and constant thickness h. The $x_1 - x_2$ plane ($x - y$ plane) of a Cartesian system coincides with the middle plane of the plate, whereas the x_3 axis (z axis) is normal to the plane. The usual approximation of plane stress state is assumed. The anisotropic constitutive relations in contracted notation are

$$\sigma_i = Q_{ij}\epsilon_j, \qquad i,j = 1,2,6 \tag{17}$$

where $(\sigma_1,\sigma_2,\sigma_6) = (\sigma_{11},\sigma_{22},\sigma_{12})$, and the reduced stiffness matrix Q is derived from the material stiffness matrix C in the following form:

$$Q_{ij} = C_{ij} - C_{i3}C_{3j}/C_{33}, \qquad i,j = 1,2,6 \tag{18}$$

To take into account the transverse shear deformation in the higher-order plate theories, the following additional equations are incorporated:

$$\sigma_\alpha = C_{\alpha\beta}\epsilon_\beta, \qquad \alpha,\beta = 4,5 \tag{19}$$

where $(\sigma_4,\sigma_5) = (\sigma_{23},\sigma_{13})$.

Let us consider two formulations of the bifurcation buckling problem for a simple supported viscoplastic plate. They include the use of the classical theory of plates and a higher-order theory.

In the framework of the classical plate theory, the governing equation of the bifurcation buckling of an elastic anisotropic plate is given by

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = h\left(\sigma_{xx}\frac{\partial^2 w}{\partial x^2} + 2\sigma_{xy}\frac{\partial^2 w}{\partial x \partial y} + \sigma_{yy}\frac{\partial^2 w}{\partial y^2}\right) \tag{20}$$

where $w(x,y)$ is the displacement of the midplane in the z direction, and the $D_{ij} = (h^3/12)Q_{ij}$ represent the bending stiffness of the plate.

Table 1 Material constants of the commercially pure titanium: in the elastic region, isotropic with Young's modulus E and Poisson's ratio ν; in the plastic region, isotropic work hardening material

E (GPa)	ν	$D_0(s^{-1})$	Z_0(MPa)	Z_1(MPa)	m	n
120	0.34	10,000	1000	1400	350	1

The governing equations, Eq. (20), with various boundary conditions on the edges of the plate, formulates the buckling problem of an elastic anisotropic rectangular plate within CPT. An exact solution of the above buckling problem is possible in the orthotropic case where $D_{16} = D_{26} = 0$ in conjunction with (SaSa) type of boundary conditions of a compressed plate ($\sigma_{11} < 0, \sigma_{22} \neq 0, \sigma_{12} = 0$). The (SaSa) notation implies that two opposite edges ($x = 0,a$) of the plate are simply supported, whereas the conditions at the remaining two, $y = (0,b)$, are arbitrary, continuous. It is clear from Eq. (14) that the initially isotropic viscoplastic plate remains orthotropic in the plastic range, provided that the loading history causes a stress state of pure biaxial compression ($\sigma_{12} = 0$).

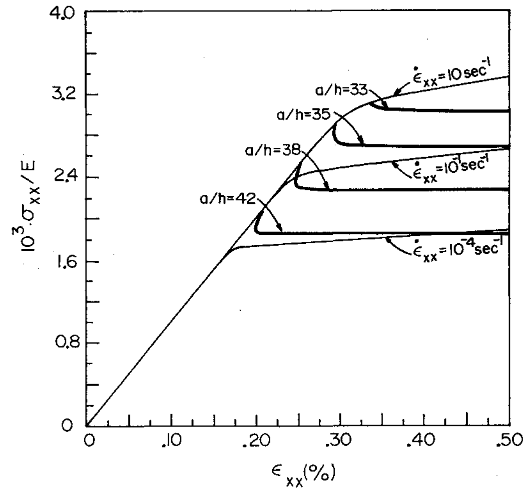

Fig. 1a Buckling curves (bold lines) of uniaxially compressed square simply supported titanium plate for various values of a/h. The fine lines represent the response of the material under uniaxial loading for several values of strain rate.

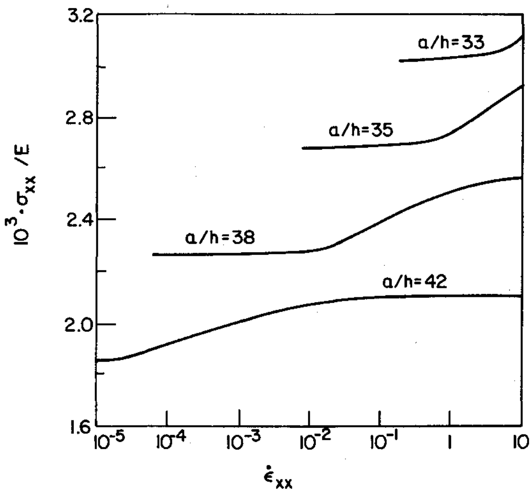

Fig. 1b Critical stress dependence on strain rate for various values of length-to-thickness ratio of the plate.

Table 2 Material constants of the aluminum alloy (2024-T4): in the elastic region, isotropic with Young's modulus E and Poisson's ratio ν; in the plastic region, isotropic work hardening material

Temperature (°C)	E(GPa)	ν	$D_0(s^{-1})$	Z_0(MPa)	Z_1(MPa)	m	n
20.0	72.4	0.33	10,000	340	435	300	10.0
148.9	69.3	0.33	10,000	340	435	300	7.0
204.4	65.7	0.33	10,000	340	435	300	4.0
260.0	56.4	0.33	10,000	340	435	300	1.6
371.1	41.5	0.33	10,000	340	435	300	0.6

An exact solution for a simply supported (SSSS) orthotropic plate

$$w(x,0) = w(x,b) = w(0,y) = w(a,y) = 0$$

$$M_x(x,0) = M_x(x,b) = M_y(0,y) = M_y(a,y) = 0 \qquad (21)$$

subjected to a uniform, biaxial state of stress

$$\sigma_{xx} < 0, \; \sigma_{yy} \neq 0, \; \sigma_{xy} = 0 \qquad (22)$$

can be written in the following form

$$w = \sum_{m,n=1}^{\infty} W_{mn} \sin\alpha x \sin\beta y \qquad (23)$$

where $\alpha = m\pi/a$ and $\beta = n\pi/b$.

The nondimensional buckling load for the mn buckling mode is given by[1]

$$\frac{\sigma_{xx} h b^2}{\pi^2 D_{22}} = -\frac{(D_{11}/D_{22})(b/a)^2 m^2 + 2(D_{12}/D_{22} + 2D_{66}/D_{22})n^2 + (a/b)^2(n^4/m^2)}{1 + (\sigma_{yy}/\sigma_{xx})(a/b)^2(n/m)^2} \qquad (24)$$

The buckling load of the corresponding viscoplastic plate can be found by using the incremental procedure described above. This is performed by checking whether Eq. (24) is satisfied at each increment of the loading history. Note that D_{ij} involve presently the instantaneous moduli.

Other available closed-form elastic solutions[1] can be similarly used for the determination of the buckling load of the corresponding viscoplastic problem. If a closed-form elastic solution is not available, numerical methods can be employed in conjunction with the present methodology.

Refined higher-order shear deformation theories of plates (HSDT) also can be used for the determination of buckling loads. These theories involve shear deformation effects and usually provide more accurate modeling of the plate behavior. A pronounced advantage of HSDT is that no shear correction factors are involved. Such a theory was presented by Reddy[13] and will be used in the present study. This theory is based on the following displacement field:

$$u_1 = u + z\,[\psi_x - (4/3)(z/h)^2(\psi_x + w_{,x})]$$

$$u_2 = v + z\,[\psi_y - (4/3)(z/h)^2(\psi_y + w_{,y})]$$

$$u_3 = w \qquad (25)$$

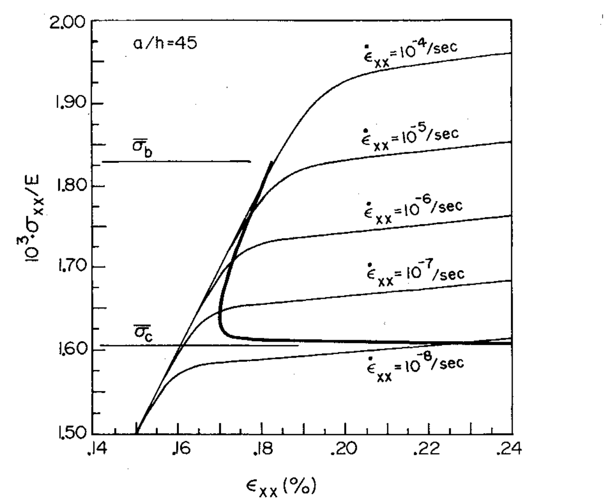

Fig. 2 Buckling curves (bold lines) of uniaxially compressed square simply supported titanium plate for $a/h = 45$. The fine lines represent the response of the material under uniaxial loading for several values of strain rate ($\bar\sigma_b = 10^3\sigma_b/E$, $\bar\sigma_c = 10^3\sigma_c/E$, where σ_b and σ_c are given in the text).

where u, v, and w are displacements of a point on the middle plane, and ψ_x and ψ_y are rotations of a line element, originally normal to the middle plane.

The following assumptions satisfy the boundary conditions of a simply supported (SSSS) [Eq. (21)] rectangular plate:

$$w = \sum_{m,n=1}^{\infty} W_{mn} \sin\alpha x \sin\beta y$$

$$\psi_x = \sum_{m,n=1}^{\infty} X_{mn} \cos\alpha x \sin\beta y$$

$$\psi_y = \sum_{m,n=1}^{\infty} Y_{mn} \sin\alpha x \cos\beta y \qquad (26)$$

where $\alpha = m\pi/a$ and $\beta = n\pi/b$.

By substituting this solution into the equilibrium equations, one obtains that the mn buckling mode is determined from the following homogeneous equations:

$$[K]\{\Delta\} = \{0\} \qquad (27)$$

where $(\Delta_1,\Delta_2,\Delta_3) = (W_{mn}, X_{mn}, Y_{mn})$. The elements $K_{ij}(i,j = 1,2,3)$ of the coefficient matrix K are given by Reddy and Phan [Eq. (16) in Ref. 14 with $\lambda = 1$].

The buckling of the corresponding viscoplastic plate is determined by the aforementioned incremental process in which the fulfillment of Eq. (27) is checked. The critical load is achieved when the matrix K (which involves the in-plane loads) becomes singular.

Applications

The proposed method is illustrated by the prediction of the critical loads of homogeneous isotropic viscoplastic plates. Two types of materials were chosen: commercially pure titanium and aluminum alloy. In the first case, the material (titanium) is highly rate sensitive. The aluminum alloy, on the other hand, is almost rate insensitive at room temperature. These materials will clearly exhibit the effect of loading rate, rate sensitivity, and temperature on the quasistatic buckling of the plates.

In Tables 1 and 2, the elastic and inelastic material constants of the titanium and aluminum alloy are given in the framework of the Bodner-Partom[10] unified viscoplastic theory. The

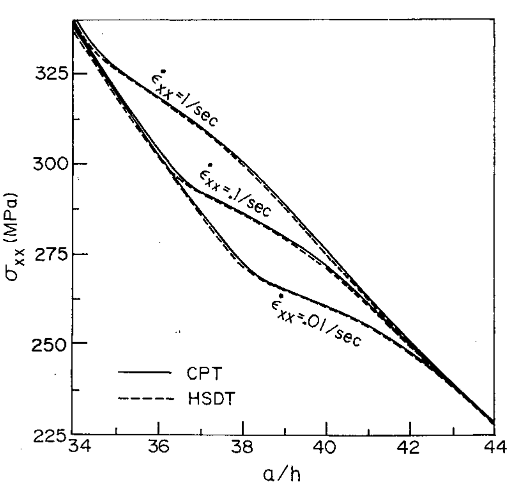

Fig. 3 Buckling stresses against length-to-thickness ratio of uniaxially compressed square simply supported titanium plate for various values of strain rate: —— CPT prediction, ----- HSDT prediction.

properties of the aluminum alloy are given at room temperature as well as at several elevated temperatures (for simplicity, the effect of thermal strain was neglected). For intermediate temperatures, the temperature-dependent parameters (i.e., E and n) have been determined by an interpolation.

The inelastic buckling of the plates is analyzed by the application of a compression stress loading σ_{xx}. For convenience, the stresses, strains, and strain rates are shown as positive in the figures.

In Fig. 1a, the critical stresses and strains of a square plate having all edges simply supported and subjected to a uniaxial compressive loading are shown for various values of length-to-thickness ratio a/h. The material of the plate is titanium, and the classical plate theory is employed. The figure exhibits two types of curves displayed by fine and bold lines. The fine lines show the stress-strain response of the titanium at various strain rates. For a spectrum of strain rates, there are many such response lines, but only three of them are shown in the figure. The bold lines shown in the figure exhibit the loci of critical points (each such point corresponds to a specific strain rate) at which buckling of the plate occurs. Each bold line (buckling curve) shown in the figure corresponds to a given value of a/h. For a given length-to-thickness ratio, a typical buckling curve was generated by computing the critical load of the plate for a specified strain rate, and various values of strain rates generate this curve. It is clearly seen that the critical stresses decrease with decreasing strain rate and that each buckling curve tends to an asymptotic level. This implies that for a given plate, the application of a compressive load below this asymptotic level will not lead to buckling. Furthermore, for a given plate one can determine a specific value of applied strain rate below which buckling will not occur. Thus, for a specific a/h one can obtain the critical (asymptotic) loading level as well as the critical strain rate for bifurcation buckling. Figure 1a clearly exhibits the significant effect of material rate sensitivity on inelastic buckling. For $a/h = 35$, for example, the strain rate change from 10/s to 0.1/s reduces the buckling stress by about 8%. To directly exhibit the dependence of the critical stress on the strain rate, Fig. 1b presents this relationship for various values of a/h. The curves show clearly the decrease of the critical stresses with decreasing strain rates.

Consider a square simply supported inelastic plate where the bifurcation buckling occurs at the elastic region at stress σ_b. Suppose that the applied loading does not exceed σ_b. After some time, it is possible that due to creep effects the material will yield and plastic flow will take place. Consequently, a plastic buckling can be obtained if the applied load is sufficiently close to σ_b. The situation is exhibited in Fig. 2 for a plate made of titanium with $a/h = 45$, using CPT. Thus, the possibility of a given plate buckling can be entirely excluded if elastic buckling of plates in the framework of rate-independent plasticity. The present results, on the other hand, include the effect loadings applied at various strain rates. It should be noted that for low values of a/h, buckling occurs at high strains for which the present analysis of infinitesimal deforma- the applied load is below the corresponding asymptotic level σ_c that is off the buckling curve. The difference between σ_b and the asymptotic level σ_c can be significant. In the case illustrated by Fig. 2, the difference is about 12%.

Figure 3 demonstrates a comparison between the critical loads of simply supported square plates made of titanium as predicted by CPT and HSDT compressed at various values of strain rates. It can be readily seen that the shear deformation effects present in HSDT have little influence on the value of the critical load even for relatively low values of length-to-thickness ratio. Similar weak influence of shear effects on the buckling stress was obtained by Shrivastava[6] who studied inelastic buckling of plates in the framework of rate-independent plasticity. The present results, on the other hand, include the effect loadings applied at various strain rates. It should be noted that for low values of a/h, buckling occurs at high strains for which the present analysis of infinitesimal deformations is not applicable. On the other hand, high values of a/h lead to buckling of the plate in the elastic region (before plastic deformations take place).

Figure 4 exhibits the buckling curves predicted by CPT at various temperatures in a simply supported square plate made of aluminum alloy and compressed at constant strain rate of 0.01/s. It can be seen that the effect of elevated temperatures is very similar to that of strain rate changes in a material with appreciable rate sensitivity.

The effect of rate sensitivity of the material can be further studied by displaying the buckling stresses obtained from CPT and HSDT for a simply supported square plate made of aluminum alloy with various values of the rate sensitivity parameter n (note that the parameter n depends on the temperature). Figure 5 exhibits the buckling stresses (normalized with respect to the temperature dependent Young's modulus) of five square plates ($a/h = 30,31,32,33,34$) as a function of this parameter; the strain rate in the length direction x is constant, $\dot{\epsilon}_{xx} = 10^{-2}$/s. The results show the decreasing of the normalized critical stress with increasing of material rate sensitivity (decreasing of n). The curves obtained by HSDT almost coincide with the corresponding curves obtained by CPT and are not shown in the figure.

It should be mentioned that the results given in the previous figures were generated by assuming that the plates were subjected to uniaxial compressive loadings applied at various values of constant strain rates. If a compressive loading at specified values of stress rate is applied, one can determine

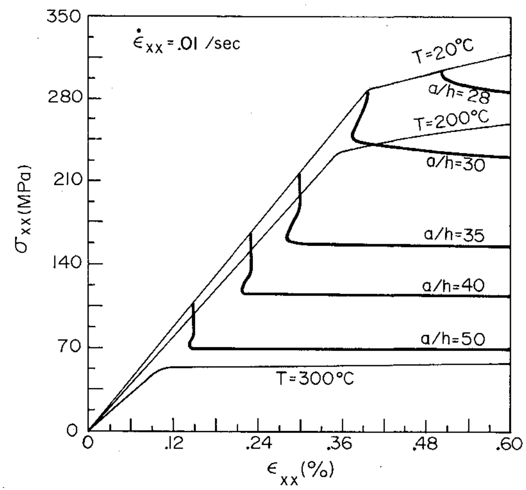

Fig. 4 Buckling curves (bold lines) of uniaxially compressed square simply supported aluminum alloy plate for various values of a/h. The fine lines represent the response of the material under uniaxial loading for several values of temperature.

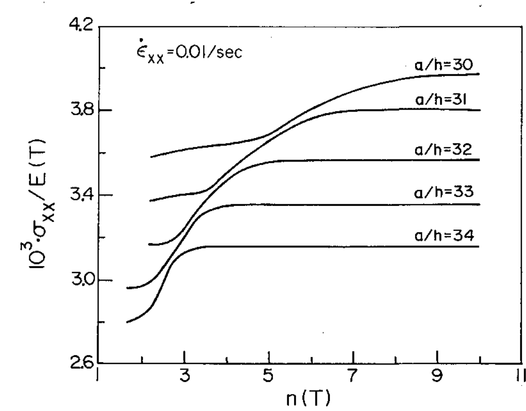

Fig. 5 Buckling stresses against material strain-rate sensitivity of uniaxially compressed square simply supported aluminum alloy plate for various values of a/h: $\dot{\epsilon}_{xx} = 0.01$/s.

from Eq. (12) the corresponding value of strain rate at each load increment. Consequently, the analysis is reduced to a loading controlled by the strain rate as discussed before.

Conclusions

A simple method is presented for the determination of the quasistatic bifurcation buckling of inelastic rate sensitive plates. The method uses the buckling solution of the corresponding perfectly elastic anisotropic structure. It is shown that the shear deformation effects are small in the cases studied in the present work. The rate sensitivity of the material, on the other hand, appears to be significant. The method can be essentially applied for various types of inelastic structures (e.g., shells) as long as the solution of the corresponding elastic problem is available. The proposed methodology provides the instability point of the structure notwithstanding whether the material state is in the elastic or plastic region. This follows from the utilization of a unified elastic-viscoplastic theory, a recent review of which can be found in Ref. 15.

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